a very useful reference book with many results which have not appeared in a book form yet. It is an important addition to the literature.

Some 1200 references have been cited, including preprints. The references appear at the end of each chapter. At the end of the book a symbol index and a name and subject index are included.

The first chapter reviews some of the classical results on polynomials of one and several variables. The second chapter provides an account of some selected inequalities involving algebraic and trigonometric polynomials. The third chapter studies zeros of polynomials with emphasis on the distribution of the zeros of algebraic polynomials, the Sendov–Iliev conjecture, as well as bounds for the zeros and the number of zeros in certain domains. The Eneström–Kakeya theorem and its various generalizations are also considered. Chapter 4 treats inequalities for trigonometric sums. In addition to classical results, special emphasis is given to the analysis of positivity and monotonicity of certain trigonometric sums and some related orthogonal polynomial sums. The fifth and sixth chapters are devoted to extremal problems for polynomials. In Chapter 5 the classical inequalities of Chebyshev, Markov, Remez, Nikolskii, Bernstein, Korkin, and Zolotarev are studied, which are basic in approximation theory. Results discussed in this chapter also include polynomials with minimal L_p norms, various generalizations of such polynomials, estimates for the coefficients of polynomials, Szegő's extremal problem, incomplete polynomials introduced by G. G. Lorentz, and weighted polynomial inequalities. Chapter 6 presents various Markov- and Bernstein-type inequalities, which are essential in proving inverse theorems of approximation. Markov- and Bernstein-type inequalities for various classes of polynomials with constraints are also presented in detail. The final chapter provides some selected applications of polynomials.

TAMÁS ERDÉLYI

A. L. LEVIN AND D. S. LUBINSKY, *Christoffel Functions and Orthogonal Polynomials for Exponential Weights on* $[-1, 1]$, Memoirs of the American Mathematical Society **535**, Amer. Math. Soc., Providence, RI, 1994, xiii + 146 pp.

For orthogonal polynomials on a bounded interval (without loss of generality we can take $[-1, 1]$), a very nice theory was developed by G. Szegő, and later generalized by Kolmogorov and Krein, in the first half of this century. Szegő's theory deals with the asymptotic properties of orthogonal polynomials on $[-1, 1]$ with an orthogonality measure μ such that the Radon–Nikodym derivative μ' is almost everywhere positive on $[-1, 1]$ and satisfies Szegő's condition

$$\int_{-1}^{1} \log \mu'(x) \frac{dx}{\sqrt{1-x^2}} > -\infty.$$

This condition implies that μ' is not allowed to be too close to zero on the interval $[-1, 1]$. Szegő's theory is very powerful, but there exist measures μ (or weights $w(x)$) on $[-1, 1]$ violating Szegő's condition, such as Pollaczek weights or weights of the form $w(x) = \exp(-(1-x^2)^{-\alpha})$, with $\alpha \geqslant 1/2$. The present monograph gives an extension of the Szegő theory for such weights, in particular for weights $w(x) = e^{-2Q(x)}$, where Q is even and convex in $(-1, 1)$ and grows sufficiently rapidly near ± 1. This means that the monograph under review deals with non-Szegő weights where the violation of Szegő's condition is near the end of the interval $[-1, 1]$. The essential point is that all interesting features of the weighted polynomials $\sqrt{w(x)}\, p_n(x)$ occur on the Mhaskar-Saff interval $[-a_n, a_n] \subset [-1, 1]$, where a_n is a sequence of numbers tending to one, determined by the weight Q. A careful analysis of the orthogonal polynomials on $[-a_n, a_n]$, rather than on $[-1, 1]$, then gives the relevant results presented by the authors. They obtain upper and lower bounds for Christoffel functions, bounds for the orthonormal polynomials, bounds for L_p norms of orthonormal polynomials, and estimates for the spacing of their zeros. The bounds hold uniformly on the whole interval $(-1, 1)$, thus not only asymptotically almost everywhere or locally uniformly on closed subsets of $(-1, 1)$. Clever use is made of logarithmic potential theory, including a chapter on the discretization of a potential. In this respect the reviewer also wants to draw attention to the recent monograph of V. Totik: *Weighted approximation with varying weight* (see the book review in this issue), who also uses logarithmic potential theory and discretization of potentials to construct appropriate approximations which can be used to obtain bounds for orthogonal polynomials.

This is a long research paper which contains important ideas and which will be essential to anyone interested in the analysis of orthogonal polynomials, in particular in the asymptotic theory for orthogonal polynomials on $[-1, 1]$ not satisfying Szegő's condition.

WALTER VAN ASSCHE

F. ALTOMARE AND M. CAMPITI, *Korovkin-type Approximation Theory and Its Applications*, de Gruyter Studies in Mathematics **17**, Walter de Gruyter, Berlin, 1994, xi + 627 pp.

If one was to compile a list of *Famous Theorems in Approximation Theory*, then one would have to include P. P. Korovkin's theorem about the convergence of positive linear operators. The statement of the theorem shocks the newcomer to approximation theory, the power of the theorem is obvious, and its proof is elegant.

Since the publication of this result by Korovkin in 1953, there have been many developments which refine or generalize Korovkin's original classical theorem. Numerous review articles and books in the area have been published. The main purpose of this new book by Altomare and Campiti is, in the words of the authors, to present "*a modern and comprehensive exposition of the main aspects of the theory in spaces of continuous functions (vanishing at infinity, respectively) defined on a compact set (a locally compact space, respectively) together with its main applications.*"

The plan of the book is surprising. I expected that the book would begin with Korovkin's original theorem, show a few well-known applications, and then branch out into more general settings. But, instead, it opens with a lengthy summary (73 pp.) of relevant aspects of Radon measures, locally convex spaces, probability theory, and stochastic processes. This introduction prepares us for Chapter 2 which outlines Korovkin-type theorems for bounded, positive Radon measures on a locally compact Hausdorff space. This chapter concludes with a discussion of Choquet boundaries and their relation to Korovkin-type theorems. Chapter 3 continues in the abstract vein established in the preceding chapter. Results in Chapter 3 are centered on the convergence of equicontinuous nets of positive linear operators to some fixed, positive linear operator (not necessarily the identity operator). Chapter 4 has applications in mind because it deals with convergence of nets of positive linear operators to the identity operator. This chapter introduces the notion of Korovkin closure of a set (of test functions) which is central to the applications of the general theory to classical problems in approximation by positive linear operators. This chapter contains Korovkin's famous $\{1, x, x^2\}$-theorem. Chapter 5 is entitled "Applications to Positive Approximation Processes on Real Intervals." Here we see applications of Korovkin's theorem to the study of approximation by positive linear operators associated with names such as Bernstein, King, Baskakov, Stancu, Cheney–Sharma, Gauss–Weierstrass, Hermite–Fejér (called Fejér–Hermite in this book), Szász–Mirakjan, Mastroianni, and others. A very interesting part (pp. 283–293) of this chapter is devoted to discussing the general relationship between positive approximation processes and probabilistic methods. The final chapter of the book deals with applications of the convergence of sequences of positive linear operators to some aspects of the theory of partial differential equations and to the theory of Markov processes.